X. PHYSICAL ACOUSTICS*

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A. RADIATION OF SOUND FROM A RANDOM FLUID VELOCITY DISTRIBUTION

To illustrate the type of investigation with which we are dealing, let us consider the radiation from a particle velocity distribution specified over a finite portion of an infinite plane. Let the particle velocity perpendicular to the plane be $u = u(\vec{r}_1, t)$, where $\vec{r}_1$ is the position vector of the source point. The radiated sound pressure in the far field can then be expressed as

$$p(\vec{r}, t) = \frac{1}{4\pi} 2\rho \int \frac{u'(\vec{r}_1, t - |\vec{r}-\vec{r}_1|/c)}{|\vec{r}-\vec{r}_1|} dS_1 \tag{1}$$

in which $u' = du/dt$, $\vec{r}$ is the position coordinate of the point of observation, ρ is the density, and dS_1 is the surface element corresponding to the source coordinate $\vec{r}_1$. The autocorrelation function for the sound pressure is $\psi(\tau) = \langle p(\vec{r}, t)p(\vec{r}, t-\tau)\rangle$, where the brackets denote a time average. From the autocorrelation function we can obtain both the power spectrum of the sound field intensity (the Fourier transform of the autocorrelation function) and the total intensity of the sound field. The total intensity is simply $I = \psi(0)/\rho c = \langle p^2\rangle/\rho c$.

From Eq. 1 we now obtain for the autocorrelation function of the sound pressure

$$\psi(\vec{r}, \tau) = \langle p(\vec{r}, t)p(\vec{r}, t-\tau)\rangle$$

$$= \frac{\rho^2}{4\pi^2} \iint \frac{\left\langle u'(\vec{r}_1, t - |\vec{r}-\vec{r}_1|/c)u'(\vec{r}_2, t - |\vec{r}-\vec{r}_2|/c - \tau)\right\rangle}{|\vec{r}-\vec{r}_1|\,|\vec{r}-\vec{r}_2|} dS_1 dS_2 \tag{2}$$

where $\vec{r}_1$ and $\vec{r}_2$ are position coordinates in the radiating surface. The autocorrelation function, then, is determined by the space-time correlation of particle acceleration in the radiating surface,

$$K(\vec{r}_1, \vec{r}_2, t_1, t_2) = \left\langle u'(\vec{r}_1, t_1)u'(\vec{r}_2, t_2)\right\rangle$$

where $t_1 = t - |\vec{r}-\vec{r}_1|/c$, and $t_2 = t - \tau - |\vec{r}-\vec{r}_2|/c$. Thus, in the far field, where $\vec{r} \gg \vec{r}_1$, we get

$$\psi(\vec{r}, \tau) = \frac{\rho^2}{4\pi^2} \frac{1}{r^2} \iint K(\vec{r}_1, \vec{r}_2, \vec{r}, \tau)\, dS_1 dS_2 \tag{3}$$

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If the time dependence is stationary, the correlation function depends only on the time difference $t_1 - t_2 = \tau - \left(|\vec{r}-\vec{r}_1| - |\vec{r}-\vec{r}_2|\right)/c$, which, when $\vec{r} \gg \vec{r}_1$, can be approximated by

$$t_1 - t_2 = \tau - \frac{(\vec{r}_2 - \vec{r}_1)\cdot\vec{r}}{rc} = \tau - \frac{\vec{R}\cdot\vec{r}}{rc}$$

where

$$\vec{R} = \vec{r}_2 - \vec{r}_1$$

If the spatial dependence of the correlation function is homogeneous, it depends only on the difference $\vec{R} = \vec{r}_2 - \vec{r}_1$. Then, if we assume that the correlation function can be separated into a product of functions depending on $\vec{R} = \vec{r}_2 - \vec{r}_1$ and $t_1 - t_2 = \tau - \tau_o$ (where $\tau_o = \vec{R}\cdot\vec{r}/rc$), we have

$$K = F(\vec{R})\, G(\tau-\tau_o)$$

In performing the integration in Eq. 3 we change the variables to

$$\vec{R} = \vec{r}_2 - \vec{r}_1$$

$$\vec{X} = \vec{r}_2 + \vec{r}_1$$

and obtain

$$\psi(\vec{r},\tau) = \frac{\rho^2}{4\pi^2}\,\frac{1}{r^2}\int d\vec{X}\int K(\vec{R},\tau-\tau_o)\, d\vec{R} \tag{4}$$

where $d\vec{X}$ and $d\vec{R}$ are the surface elements corresponding to the variables $\vec{X}$ and $\vec{R}$. The surface element $d\vec{R}$ is $RdRd\phi_1$, and if the velocity distribution is "isotropic," the function $F(\vec{R})$ is independent of the direction of $\vec{R}$ and depends only upon the magnitude of $\vec{R}$. Furthermore, if the correlation function is different from zero over only a relatively small distance, L, which is much smaller than the size of the radiating area, we need not be concerned with the complications arising at the boundary of the source region when we perform the integration over $\vec{R}$. In other words, we integrate over one of the characteristic patches (inside which the correlation function is essentially different from zero), and then we add up the contributions from the various patches by integration over $\vec{X}$. This integration merely involves multiplying by the area of the source region.

If the time dependence is harmonic, we have

$$G(\tau-\tau_o) = \exp[-i\omega(\tau-\tau_o)]$$

The integral over the variable $\vec{R}$ is then of the form

$$I(\theta) = \int F(R)\, RdR \int_0^{2\pi} \exp(+ikR \sin\theta \cos(\phi-\phi_1))\, d\phi_1$$

$$= 2\pi \int F(R)\, J_o(kR \sin\theta)\, RdR$$

In the particular case of a Gaussian correlation function of the form

$$F(R) = \langle u'^2 \rangle \exp(-R^2/L^2)$$

the integral can be written as

$$I(kL \sin\theta) = 2\pi L^2 \langle u'^2 \rangle \int_0^{\infty} \exp(-x^2)\, J_o(xkL \sin\theta)\, xdx$$

$$= \pi L^2 \langle u'^2 \rangle \exp[-(kL \sin\theta)^2]$$

The correlation function for the sound pressure at the point of observation is, then,

$$\psi(\tau) = \exp(-i\omega\tau) \frac{\rho^2}{4\pi^2} S\pi L^2 \langle u'^2 \rangle \exp[-(kL \sin\theta)^2]$$

and the acoustic intensity at that point is

$$\frac{\psi(0)}{\rho c} = SL^2 \langle u'^2 \rangle \frac{\rho}{4\pi c} \exp[-(kL \sin\theta)^2]$$

which for harmonic time dependence becomes

$$\frac{\psi(0)}{\rho c} = S(kL)^2 \langle u'^2 \rangle \frac{\rho c}{4\pi} \exp[-(kL \sin\theta)^2]$$

Notice that the sound intensity is proportional to the size of the patch, L^2.

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B. RADIATION OF SOUND FROM MOVING SOURCES

Blokhintzev (1) has investigated the radiation of sound from a source moving with a constant velocity that may or may not exceed the speed of sound. The source is considered to be a monopole point source. However, in his book the strength of the monopole was specified in terms of the acceleration of the mass flow emanating from the source. The standard way of specifying the strength of the monopole is in terms of the time rate of the mass flow. Thus, if the strength of the monopole is specified in terms of t, one finds that the field radiated from the moving source not only contains a monopolelike contribution but also a dipole contribution consisting of both a near and a far field. The sound pressure from the moving source thus obtained becomes, for $M < 1$,

$$p = \frac{1}{4\pi}\left\{(1-M^2)^{3/2}\frac{F'}{R_1}\left(1+M\frac{x_1}{R_1}\right)+\frac{V}{1-M^2}\frac{x_1}{R_1^3}F\right\}$$

where c is the speed of sound; V is the source velocity; $M = V/c$; F is the monopole source strength defined by $Q(r,t) = F(t)\,\delta(x-Vt)$, in which Q is the mass flow;

$$x_1 = \frac{x - Vt}{(1-M^2)^{1/2}}$$

$$R_1 = \left(x_1^2+y^2+z^2\right)^{1/2};$$

and the prime indicates the derivative with respect to its argument.

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References

1. D. I. Blokhintsev, Acoustics of a Nonhomogeneous Moving Medium (translated from the Russian by S. Reiss), Tech. Memo. 1399, National Advisory Committee for Aeronautics, Washington, D.C., February 1956, Chap. III, pp. 74-111.

C. WAVES OF FINITE AMPLITUDE IN AN INHOMOGENEOUS MEDIUM

The object of this study is to try to extend the well-known Riemann description of propagation of finite amplitude sound waves to the case in which the medium is not isentropic. In the isentropic case the exact hydrodynamical equations governing the space-time dependence of the variables pressure, density, and particle velocity can be brought into the following Riemannian form

$$\frac{\partial\psi}{\partial t}+(u\pm c)\frac{\partial\psi}{\partial x} = 0 \tag{1}$$

with the solution

$$\psi = \begin{matrix}p\\ \rho\\ u\end{matrix} = f(x-(u\pm c)t) \tag{2}$$

where

$$c^2 = \rho^2\left(\frac{\partial u}{\partial\rho}\right)_s^2 \tag{3}$$

The local adiabatic speed of sound, c, is $\left[\left(\frac{\partial p}{\partial\rho}\right)_s\right]^{1/2}$, a function of temperature only. A point of constant ψ on the wave is propagated with velocity $u + c$ or $u - c$, depending on the direction of propagation, and therefore the waveform becomes progressively distorted. Although the stable waveform cannot be predicted from these equations because the effects of viscosity and heat conduction have been neglected, the results do provide a starting point for the study of finite-amplitude waves in homogeneous media.

Somewhat different results are obtained for an ideal fluid if the entropy is taken to be a function of position. Under these conditions, an observer in a Eulerian frame (stationary) would observe entropy fluctuations in the medium of the order of $u\frac{\partial S_o(x)}{\partial x}$, in which $\frac{\partial S_o(x)}{\partial x}$ is the entropy gradient in the medium, while an observer in a Lagrangian frame (moving with the fluid particles) would still say that the motion is isentropic.

In formulating the equations of motion in an Eulerian reference frame, one must therefore take the pressure and particle velocity to be functions of both density and entropy. The following set of equations must be used in the derivation of the exact one-dimensional wave equations:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0 \qquad \text{(continuity equation)} \tag{4}$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho u^2)=F-\frac{\partial p}{\partial x} \qquad \text{(momentum equation)} \tag{5}$$

$$p=p(\rho,s) \qquad \text{(equation of state)} \tag{6}$$

$$u=u(\rho,s) \tag{7}$$

$$\frac{\partial s}{\partial t}+u\frac{\partial s}{\partial x}=0 \qquad \text{(adiabatic condition)} \tag{8}$$

Here, F is the external force per unit volume. After a considerable amount of manipulation, these equations can be put into the following form:

$$\frac{\partial\rho}{\partial t}+\left\{u+\rho\left[\frac{\left(\frac{\partial p}{\partial s}\right)_\rho\left(\frac{\partial u}{\partial\rho}\right)_s-\left(\frac{\partial u}{\partial s}\right)_\rho\left(\frac{\partial p}{\partial\rho}\right)_s}{\left(\frac{\partial p}{\partial s}\right)_\rho-\rho^2\left(\frac{\partial u}{\partial\rho}\right)_s\left(\frac{\partial u}{\partial s}\right)_\rho}\right]\right\}\frac{\partial\rho}{\partial x}=\frac{-\rho\left(\frac{\partial u}{\partial s}\right)_\rho F}{\left(\frac{\partial p}{\partial s}\right)_\rho-\rho^2\left(\frac{\partial u}{\partial\rho}\right)_s\left(\frac{\partial u}{\partial s}\right)_\rho} \tag{9}$$

$$\frac{\partial p}{\partial t}+\left\{u+\frac{\left(\frac{\partial p}{\partial\rho}\right)_s}{\rho\left(\frac{\partial u}{\partial\rho}\right)_s}\right\}\frac{\partial p}{\partial x}=\frac{\left(\frac{\partial p}{\partial\rho}\right)_s}{\rho\left(\frac{\partial u}{\partial\rho}\right)_s}F \tag{10}$$

$$\frac{\partial u}{\partial t}+\left\{u+\rho\left(\frac{\partial u}{\partial\rho}\right)_s\right\}\frac{\partial u}{\partial x}=0 \tag{11}$$

$$\left\{\left(\frac{\partial p}{\partial\rho}\right)_s-\rho^2\left(\frac{\partial u}{\partial\rho}\right)_s^2\right\}\frac{\partial\rho}{\partial x}+\left\{\left(\frac{\partial p}{\partial s}\right)_\rho-\rho^2\left(\frac{\partial u}{\partial\rho}\right)_s\left(\frac{\partial u}{\partial s}\right)_\rho\right\}\frac{\partial s}{\partial x}=F \tag{12}$$

Equations 9, 10 and 11 are recognized to be analogous to Eq. 1, except that the right-hand side is not zero and the local speed of sound is replaced by a complicated expression that is different for pressure, density, and particle velocity. Equation 12 is the generalization of the Riemann condition for an inhomogeneous medium. For no external force and $\partial s/\partial x = 0$ it reduces to Eq. 3. Approximate solutions for these equations have not yet been found, but would be of interest in many problems. For example, these equations can, in principle, be used to find the attenuation of an "N-wave" in the atmosphere.

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D. GENERATION OF SOUND BY PARALLEL JETS

The project reported in the previous Quarterly Progress Report (1) has been completed. As indicated in that report a definite deviation from the Lighthill-Ingard (2) equation is observed.

The experiment used three parallel air jets, 0.25 inch in diameter. The orifice Mach number ranged from 0.17 to 0.26. The distance between the jets was varied, and the total acoustic power output was measured as a function of Mach number. The total acoustic power output depended on the fourth power of the Mach number.

It appears that the deviation from theoretical predictions is caused by interaction of the jets. The Reynolds number of jets was approximately 2.5×10^4. Therefore, turbulence was fully developed, and the Lighthill-Ingard equations should hold if no interaction is present.

Details of the experiment and a complete analysis of the results are available in the author's thesis (3).

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References

1. E. J. Martens, Jr., Generation of sound by parallel jets, Quarterly Progress Report No. 60, Research Laboratory of Electronics, M.I.T., Jan. 15, 1961, pp. 175-176.

2. U. Ingard, J. Acoust. Soc. Am. 31, 1202 (1959).

3. E. J. Martens, Jr., Noise Generation by Parallel Interacting Air Jets, S.B. Thesis, Department of Physics, M.I.T., January 1961.

E. INFLUENCE OF VIBRATIONS ON CONTACT FRICTION

Experiments are under way to determine the effect of a fluctuating normal force on the coefficient of static friction. The experimental program consists of superposing a time-variant normal force, $N(t)$, on the static normal force, N_o, between two bodies in contact with $N(t) < N_o$. It appears reasonable to assume that the standard relation

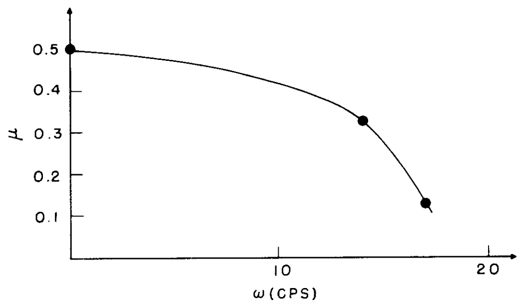

Fig. X-1. The coefficient of static friction vs. the frequency of the time-varying normal force.

$\mu = F/N_0$ would hold if the time average of N_0 plus $N(t)$ is substituted for N_0. If $\overline{N(t)} = 0$, there should be no effect of the coefficient of static friction. The preliminary experiments were performed with a wooden block that had a felt pad on its base. The block was free to slide on a smooth wooden plank, and the coefficient of static friction was measured. A small, sinusoidally varying normal force was supplied. If the approach outlined above is correct, the coefficient of static friction should be a constant. The results are displayed in Fig. X-1. They obviously contradict our theory.

A more extensive experimental program is being designed, to examine, in particular, the frequency dependence of this phenomenon. A theoretical investigation is also being carried out.

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